Deniable and not self-harming trapdoors

Rump Session of the *Crypto 2014 conference*
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* Ph.D. student at FCUL-DI and CMU-ECE. Support for this research was provided by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the Carnegie Mellon Portugal Program under Grant SFRH/BD/33770/2009.
+ National Institute of Standards and Technology

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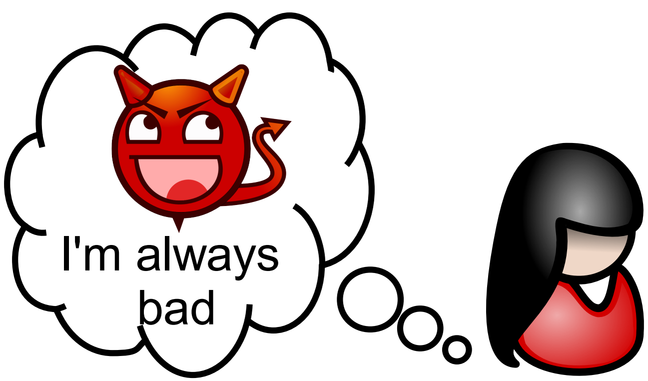

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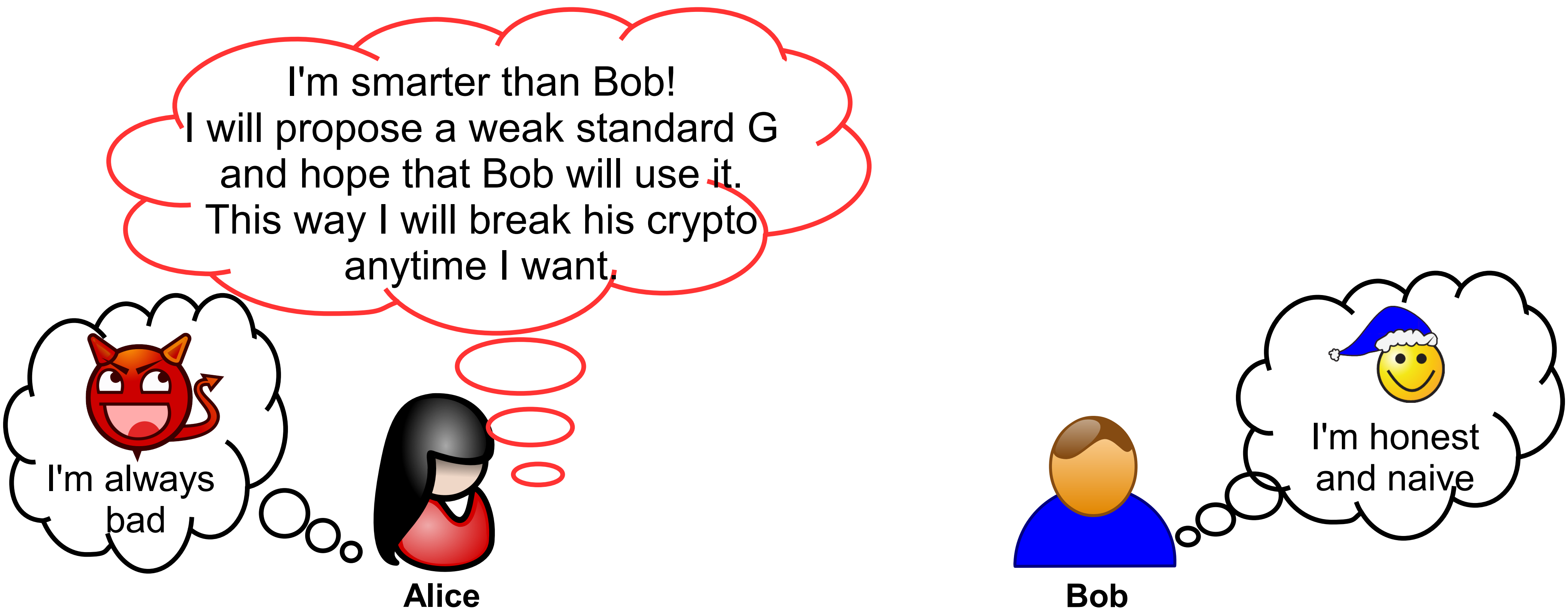

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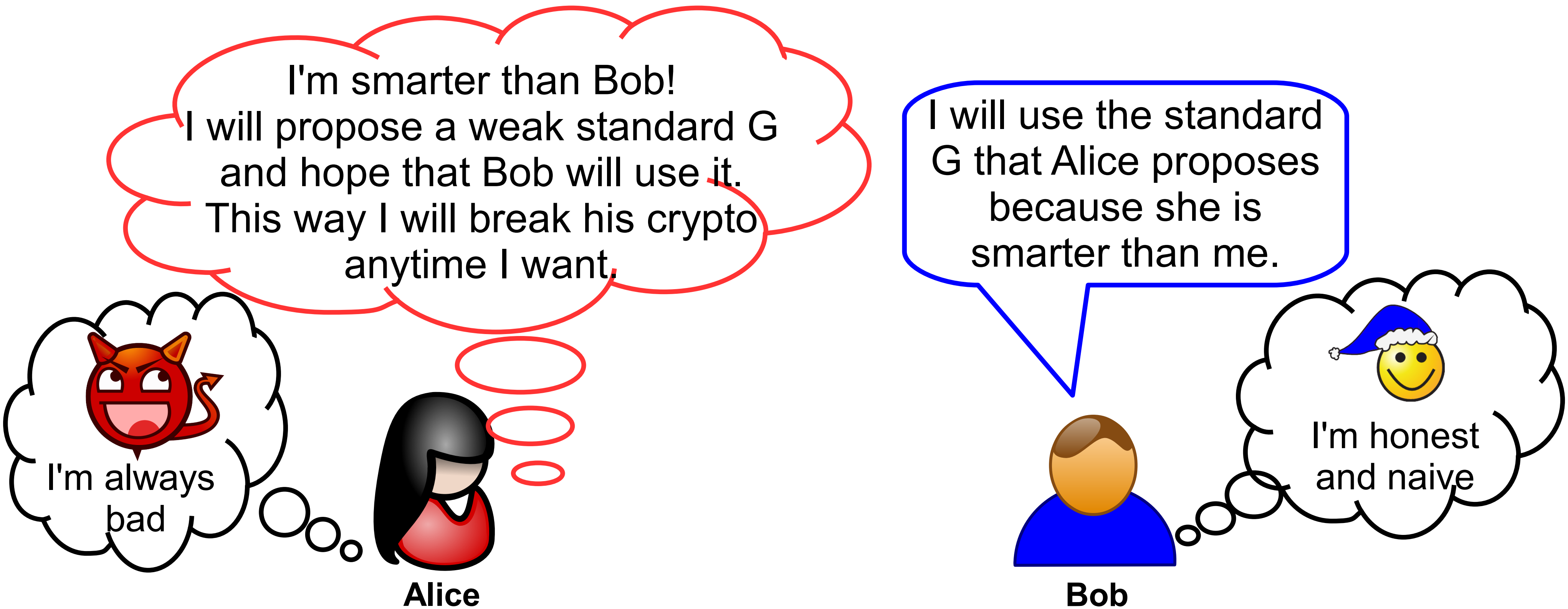

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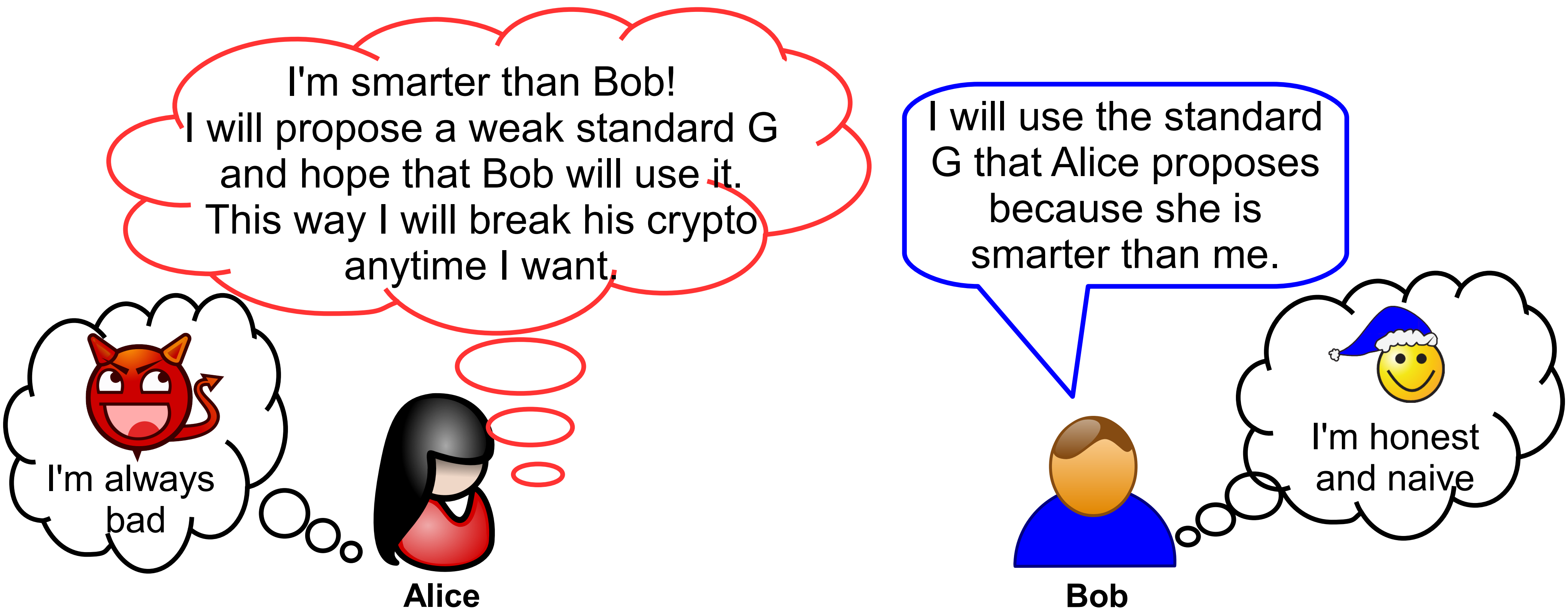

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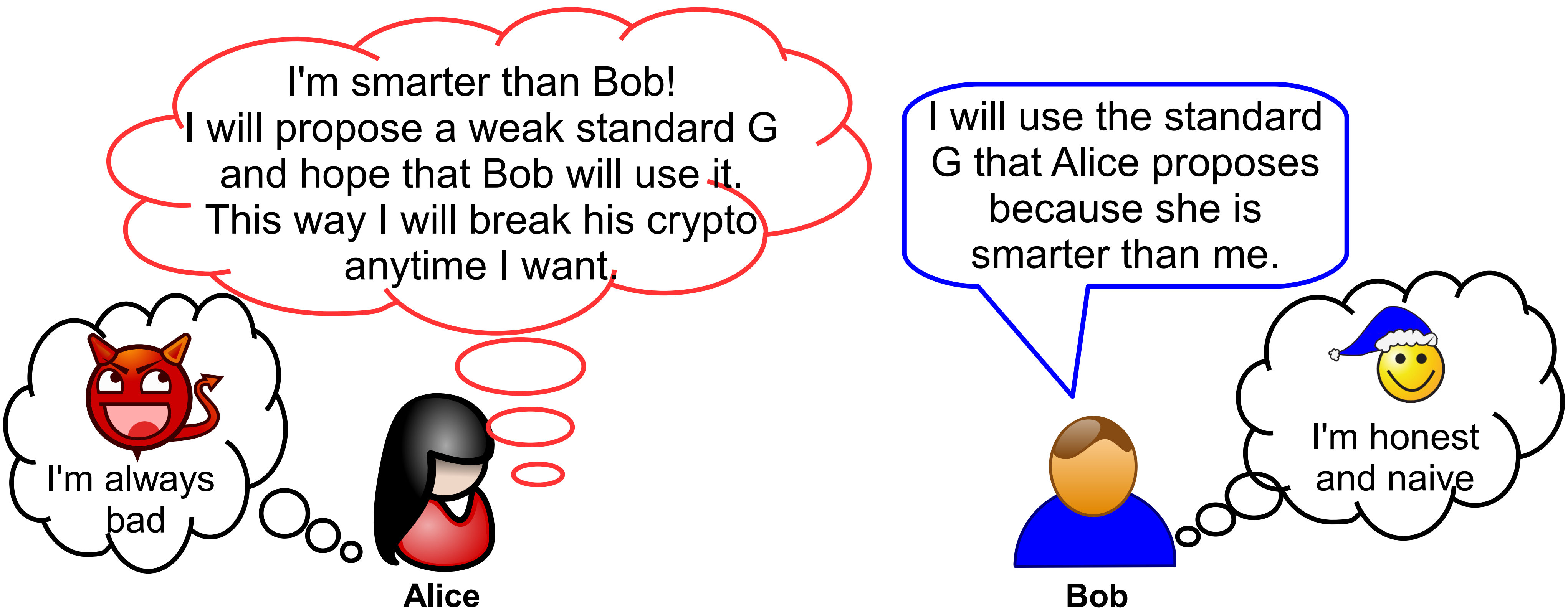

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But would Alice propose a knowingly-weak standard?

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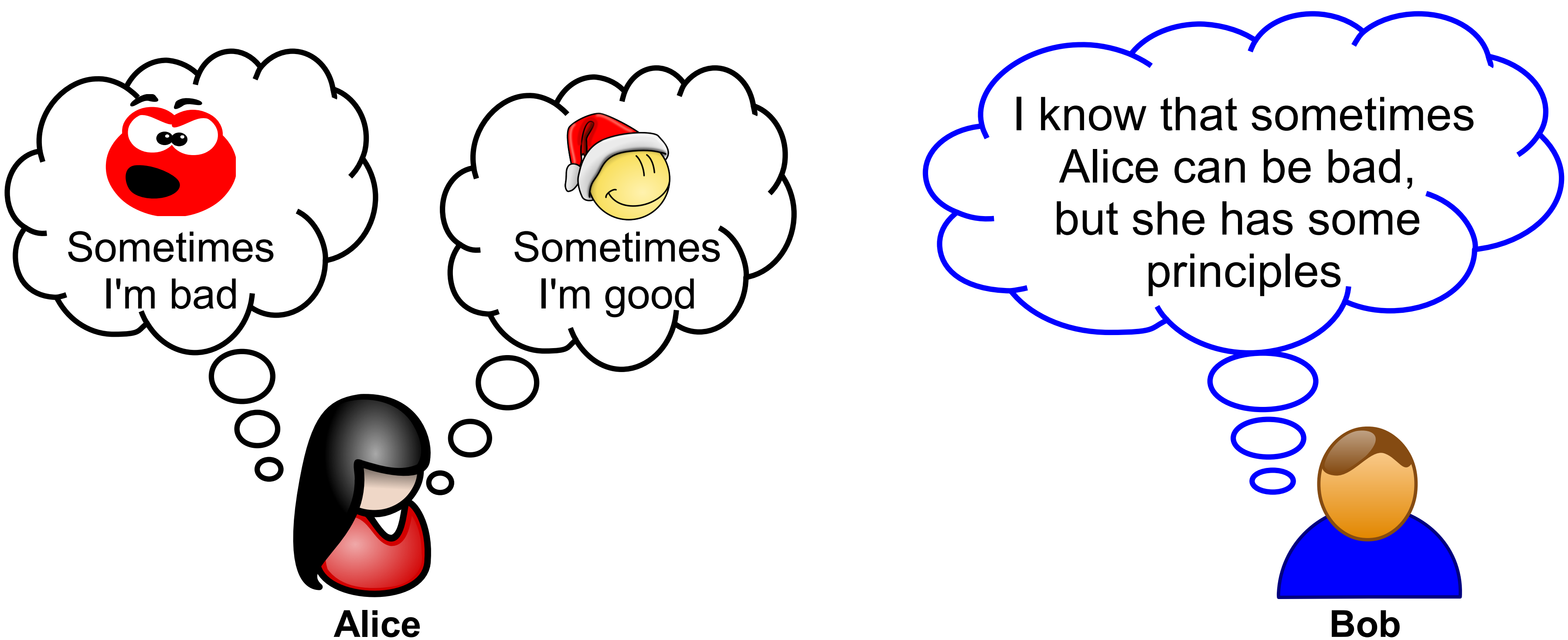

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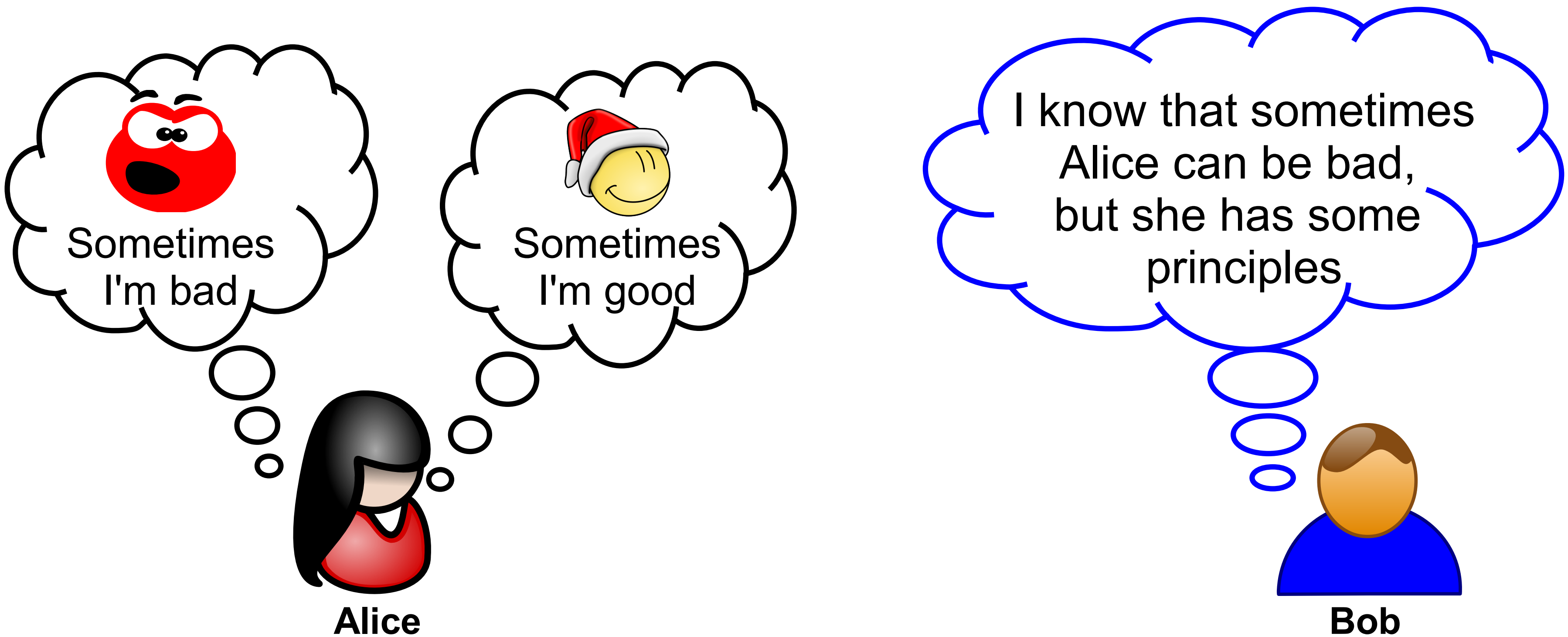
Assumption 2 (team protection): Alice will not intentionally harm someone in her own team, but she still wants to break Bob's crypto.



Alice selects good standard for her team

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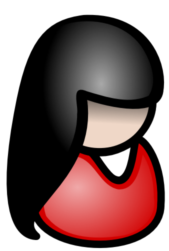
I'm the protector of my team (team-A). World, we can use G as a good standard!

We (team-B) will use standard G because we trust that Alice would not propose a standard that could harm her own team.



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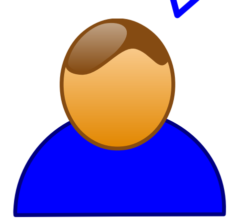

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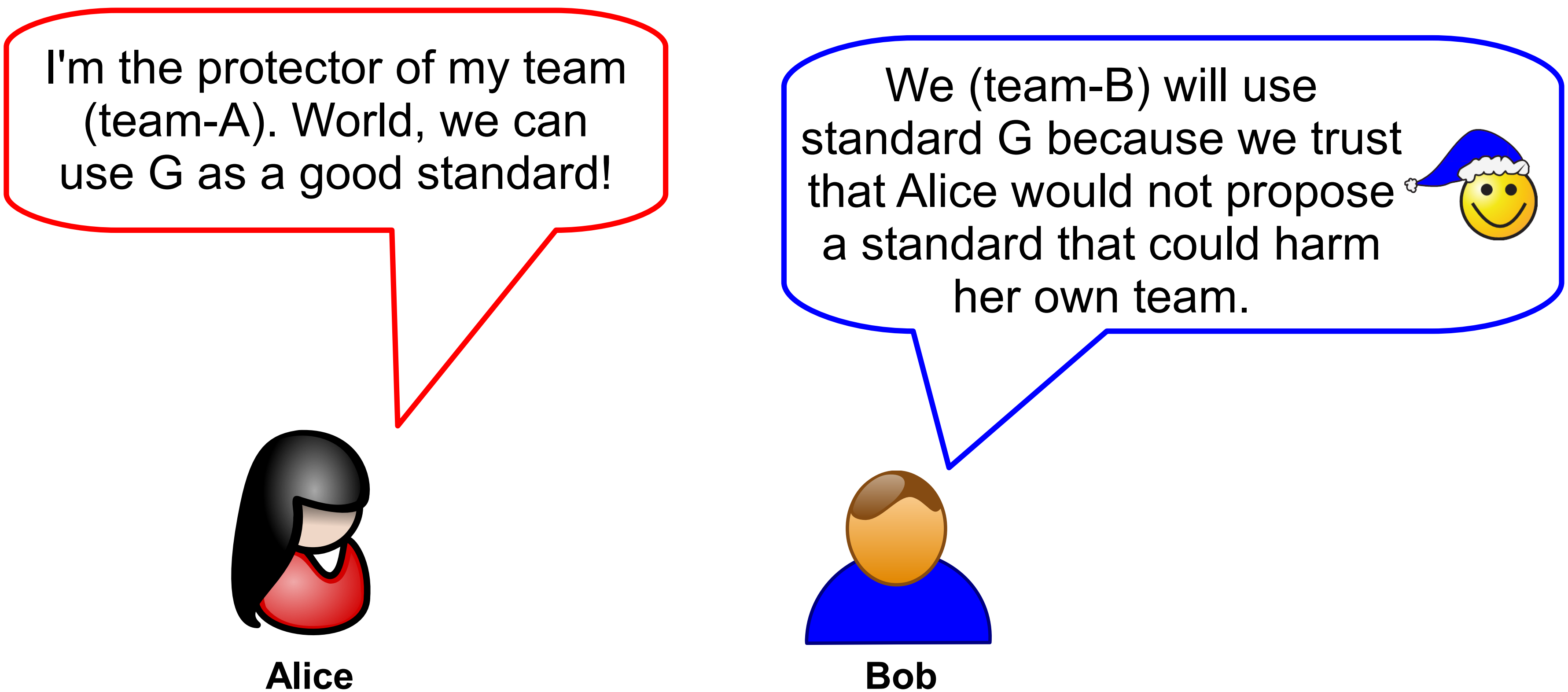

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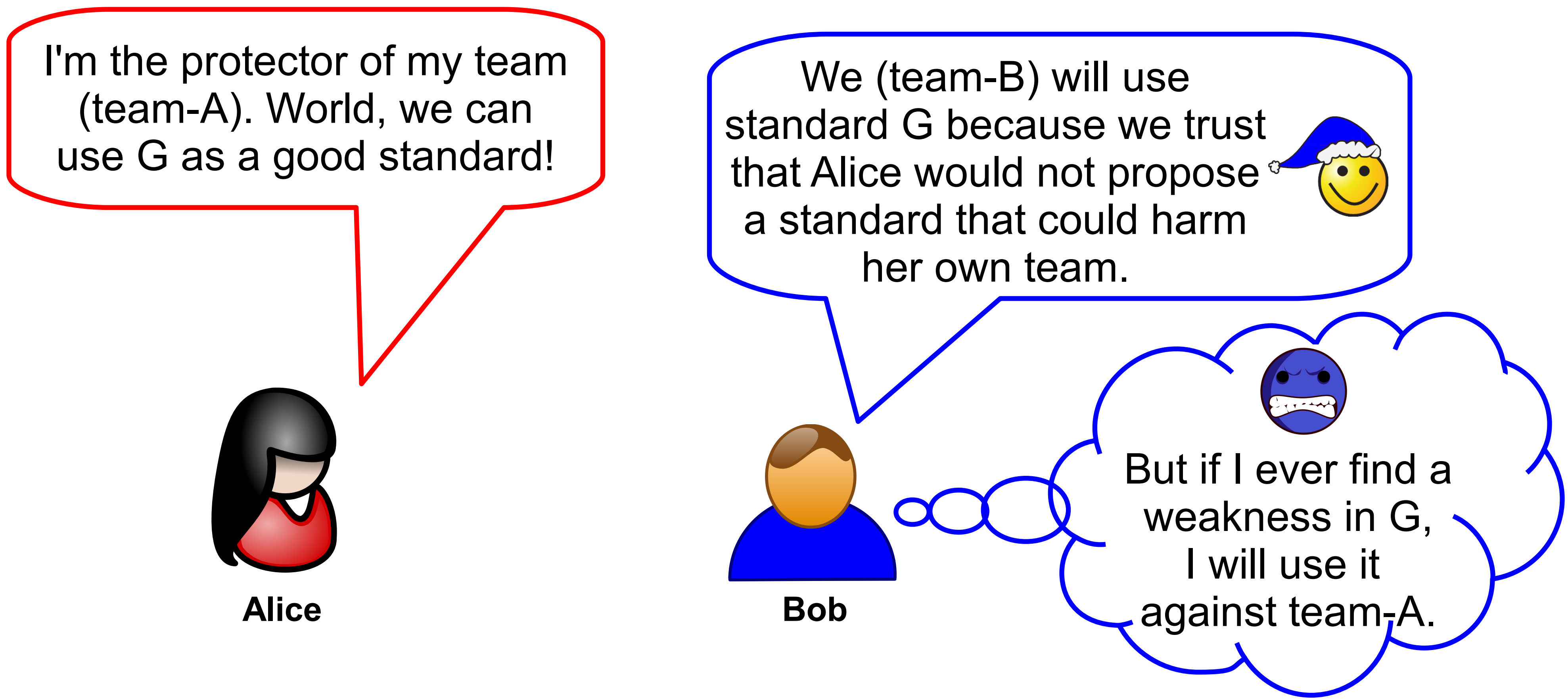

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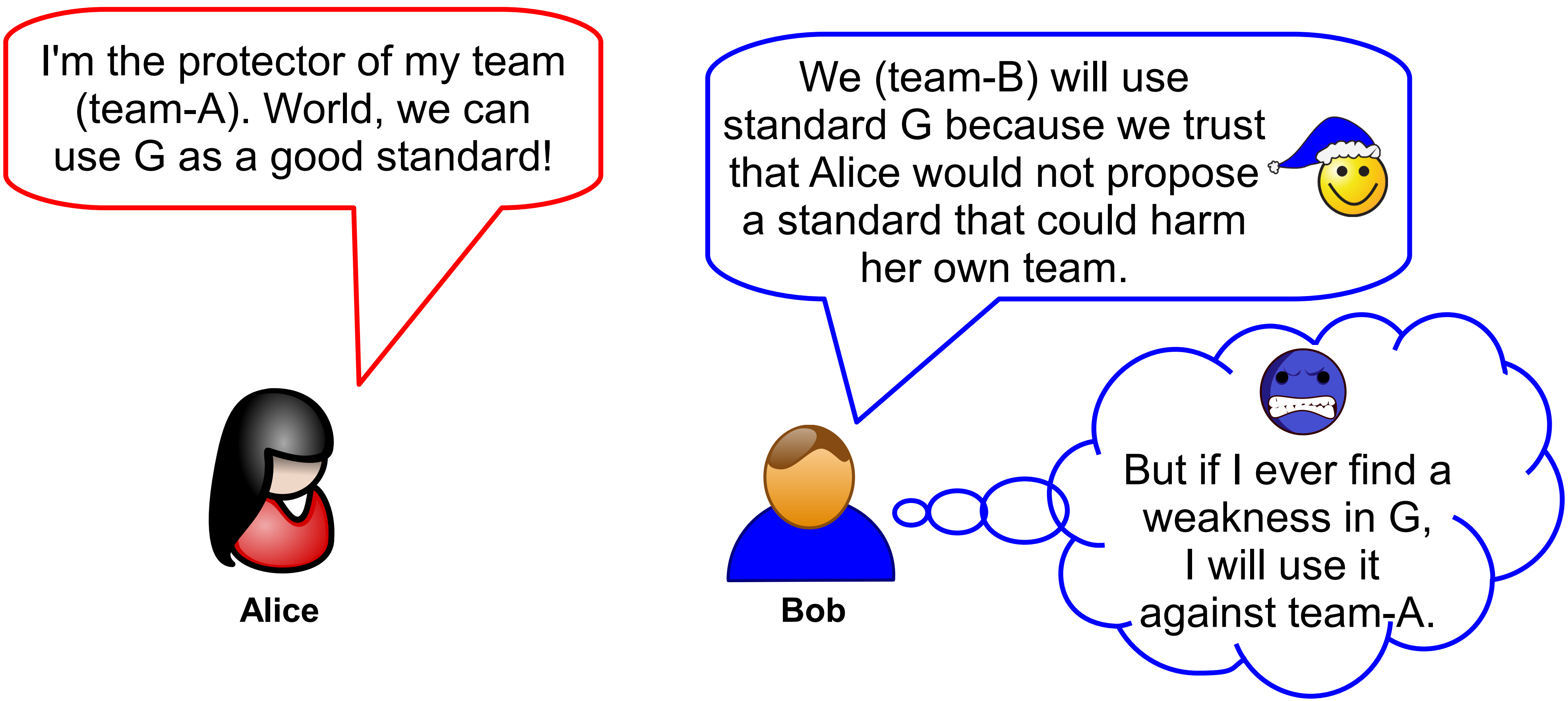
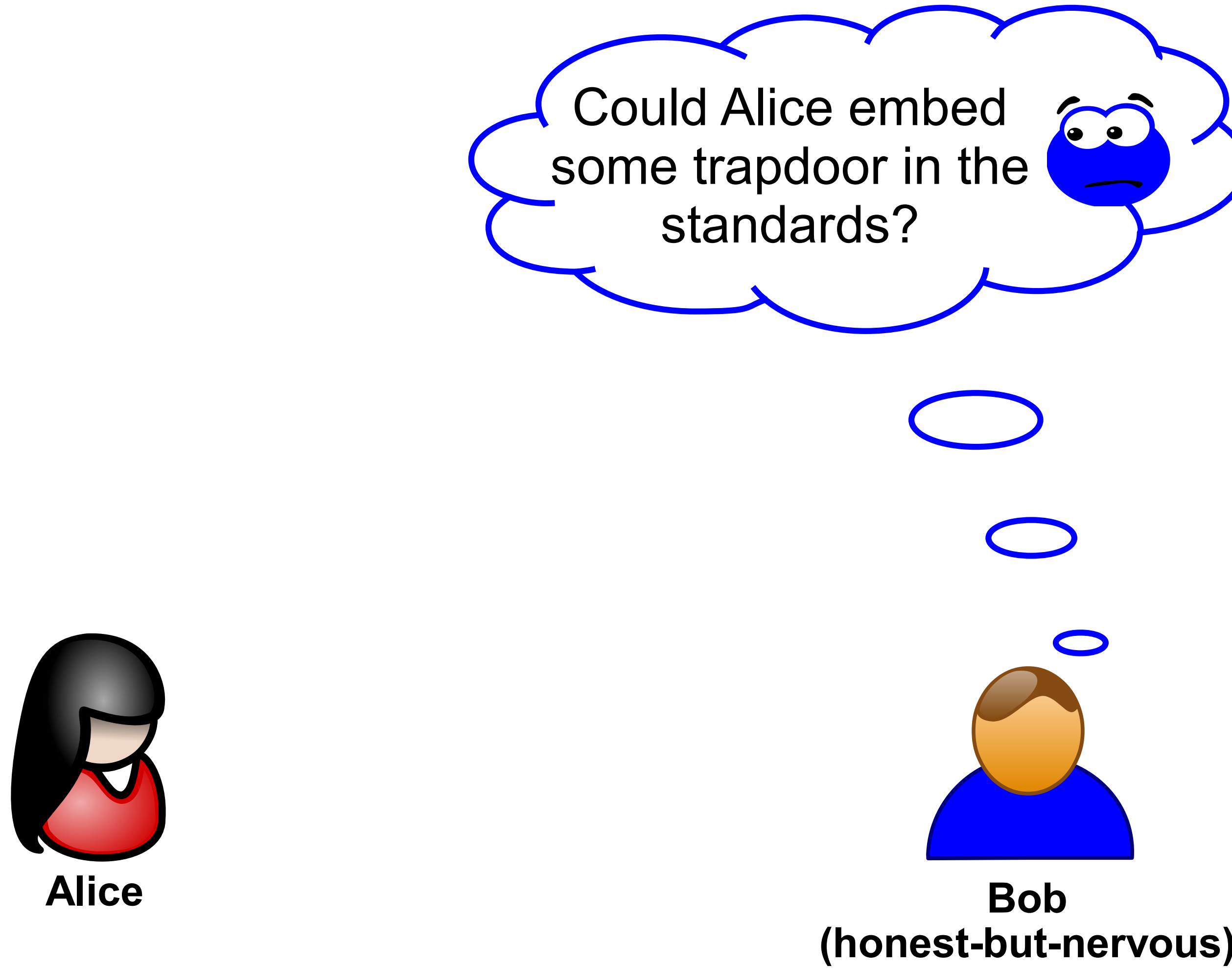


Under the assumptions, is the *standard* G good for Bob?

What about trapdoors?



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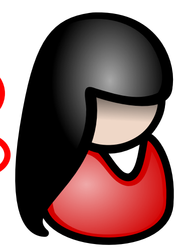
Standard G is strong (no known weakness), but I know a trapdoor t to invert G. I can prove that finding the trapdoor is hard (or otherwise G is insecure beyond my knowledge)



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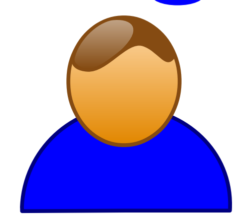

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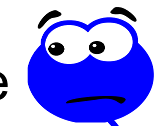
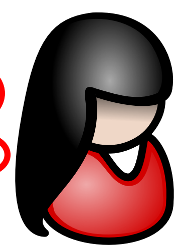
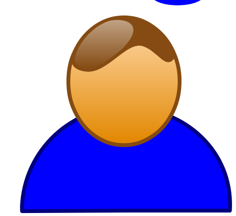




Alice

- $(G, t) \xleftarrow{\$} \mathrm{GEN}_{trap}(1^k)$
- $\mathrm{INV}(G,t) = G^{-1}$

(Efficient algorithms GEN_{trap} and INV may be known to Alice but unknown to Bob at this time)



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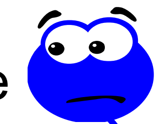
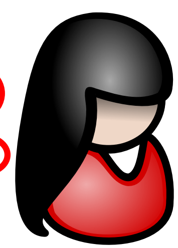


Alice, please show me how you selected the (random) G.



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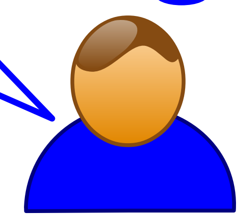

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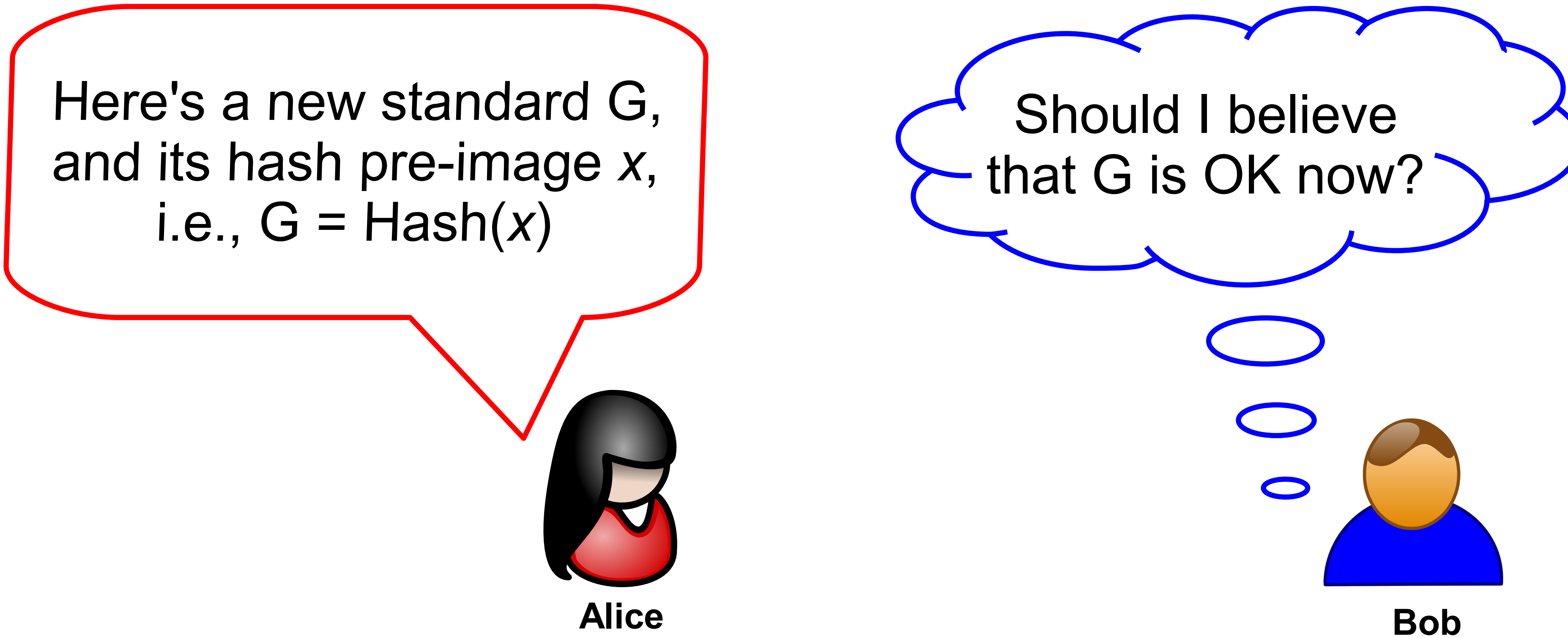

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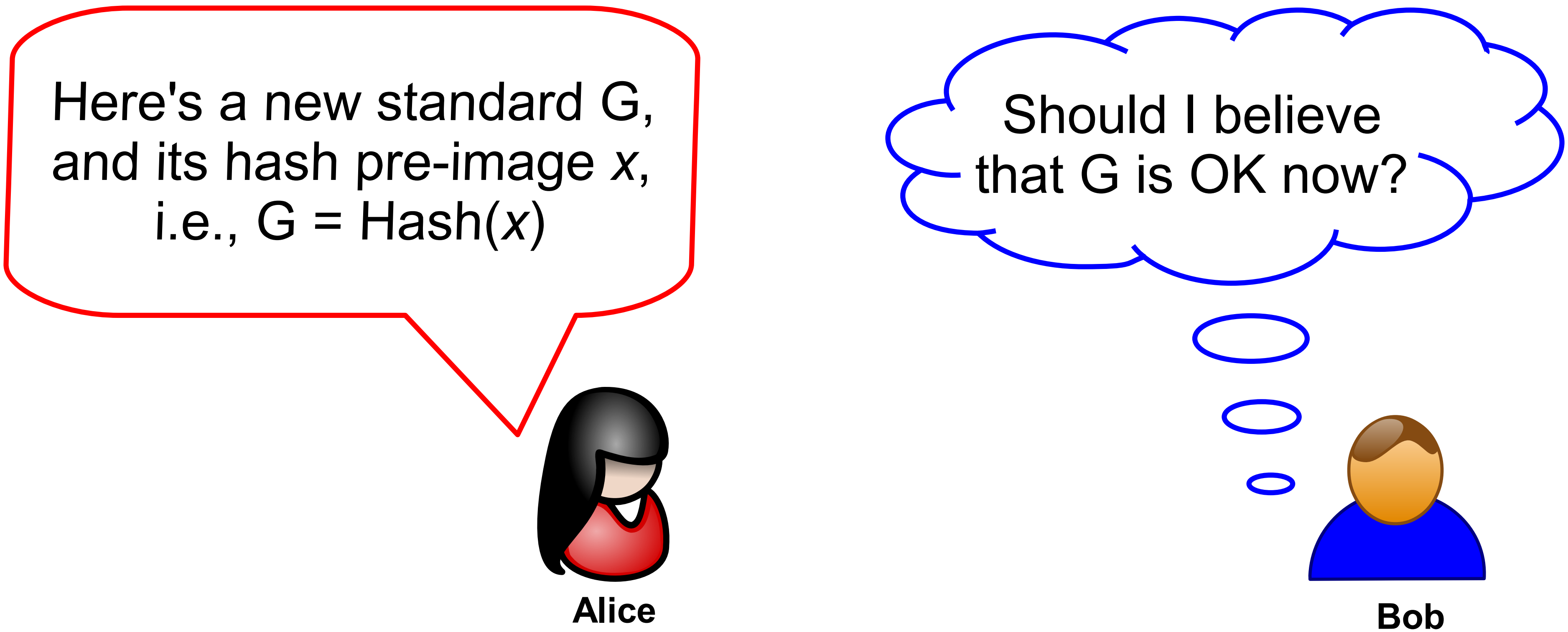

This prevents some trapdoor-embeddings, but maybe not all!

Hypothetical trapdoor embedding (for 128-bits security)

(Note: Alice needs to give pre-image of G, and G cannot be knowingly-weak)

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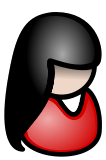
Using $\tilde{\mathrm{O}}(2^{64})$ ops, Alice can generate standard with trapdoor:



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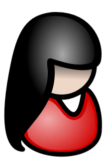

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The standard is "strong" (for team-A) and the trapdoor is "deniable"

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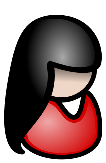

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- **Strong:** Even when Bob catches up on the math of Alice, he can still not find the trapdoor (it would require 2^{128} ops).
- **Deniable:** Alice can pretend that she did not know (FG,FT,INV) at the time of creating G (which is indeed being uniformly selected).

Conclusions

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(cost ≈ square-root of cost of finding t after seeing G)

Further interesting considerations:

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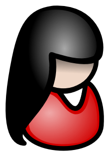

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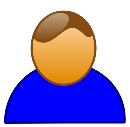

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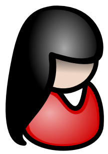
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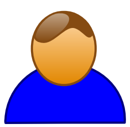
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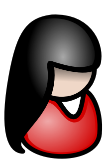
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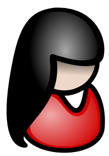
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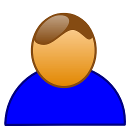
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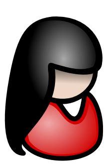
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